$$(\psi, H\psi)$$

$$\geq E^{TF}\left(N, Z, R, 6\left(\frac{\pi^2}{2}\right)^{\frac{1}{3}} - \frac{5}{3} q^{\frac{2}{3}} 1.68 a\right) - 1.68 \frac{N\alpha^2}{2a}$$

$$\geq \left(6\left(\frac{\pi^2}{2}\right)^{\frac{1}{3}} - \frac{5}{3} q^{\frac{2}{3}} 1.68 a\right)^{-1} E^{TF}(1,1,0,1) \sum_j Z_j^{\frac{7}{3}} - 1.68 \frac{N\alpha^2}{2a}$$

$$a = \frac{6\left(\frac{\pi^2}{2}\right)^{\frac{1}{3}}}{\frac{5}{3} q^{\frac{2}{3}} 1.68} \varepsilon$$

$$= -\frac{1}{6}\left(\frac{\pi^2}{2}\right)^{-\frac{1}{3}} \frac{7.356\,\cancel{\alpha}^2 \sum Z^{\frac{7}{3}}}{(1-\varepsilon)} - \frac{1.68 N \cancel{\alpha}^2}{2} \frac{\frac{5}{3} q^{\frac{2}{3}} \cancel{1.68}}{6\left(\frac{\pi^2}{2}\right)^{\frac{1}{3}}} \frac{1}{\varepsilon}$$

$$= -\frac{\alpha^2}{2\cdot 6\left(\frac{\pi^2}{2}\right)^{\frac{1}{3}}} \cdot \left(\frac{2\times 7.356 \sum_j Z_j^{\frac{7}{3}}}{1-\varepsilon} + \frac{\frac{5}{3} 1.68^2 q^{\frac{2}{3}} N}{\varepsilon}\right)$$

$$= -\underbrace{\frac{\frac{5}{3} 1.68^2 q^{\frac{2}{3}} \alpha^2 N}{2\cdot 6\left(\frac{\pi^2}{2}\right)^{\frac{1}{3}}}}\left(1 + \sqrt{\frac{2\times 7.356}{\frac{5}{3} 1.68^2 q^{\frac{2}{3}}} \frac{1}{N} \sum_j Z_j^{\frac{7}{3}}}\right)^2$$

$$= -0.23024\ldots q^{\frac{2}{3}} \alpha^2 N \left(1 + 1.768\ldots \sqrt{\frac{1}{N} \sum_j Z_j^{\frac{7}{3}}}\right)^2$$

~~5.627~~ $\boxed{5.6027 \text{ Rydbergs}}$

§7.4 Other Routes to a Proof of Stability

@ Dyson-Lenard (1967)

$$\mathcal{E} \geq -1.3 \times 10^{14} N$$

@ Federbush (1975)

constructive quantum field theory

@ Fefferman (1983)

$\mathbb{R}^3 \longrightarrow$ boxes

@ Graf (1997)

$\mathbb{R}^3 \longrightarrow$ simpl~~ex~~ices

§7.5 Extensivity of Matter

$$\mathcal{E}(\psi) \geq -{}^{\exists}A N \quad \leftarrow \text{Thm 7.1}$$

$$T\psi \geq {}^{\exists}K \int_{\mathbb{R}^3} \rho_\psi(x)^{\frac{5}{3}} dx \quad \leftarrow \text{Cor. 4.1}$$

Theorem 7.2 (Extensivity of Matter)

(i) $\forall p > 0 \quad \exists \gamma_p > 0$

s.t. $$\left(\frac{1}{N} \int_{\mathbb{R}^3} \rho_\psi(x) |x|^p dx \right)^{\frac{1}{p}} \geq \gamma_p K^{\frac{1}{2}} \left(\sqrt{\frac{E}{N} + A} + \sqrt{A} \right)^{-1} N^{\frac{1}{3}}$$

(ii) $\Omega \subset \mathbb{R}^3$

$$\frac{1}{N} \int_\Omega \rho_\psi(x) dx \leq K^{-\frac{3}{5}} \left(\sqrt{\frac{E}{N} + A} + \sqrt{A} \right)^{\frac{6}{5}} \times \left(\frac{\mathrm{vol}(\Omega)}{N} \right)^{\frac{2}{5}}$$

pf.

(i) $E = \mathcal{E}(\psi) = \lambda T\psi + (1-\lambda)T\psi + V\psi$

$$0 < \lambda < 1.$$

$$(1-\lambda)T\psi + V\psi \geq -\frac{AN}{1-\lambda}$$

$\because$) $\psi_\lambda(x) = \lambda^{\frac{3}{2}}\psi(\lambda x) \qquad (\lambda > 0)$

$$\begin{aligned} T\psi_\lambda &= \int |\nabla\psi_\lambda(x)|^2 dx \\ &= \lambda^2 \int |(\nabla\psi)(\lambda x)|^2 \lambda^3 dx \\ &= \lambda^2 T\psi \end{aligned}$$

$$\begin{aligned} V\psi_\lambda &= \int \frac{1}{|x|} |\psi_\lambda(x)|^2 dx \\ &= \lambda \int \frac{1}{|\lambda x|} |\psi(\lambda x)|^2 \lambda^3 dx \\ &= \lambda V\psi \end{aligned}$$

$$\underbrace{T\psi_\lambda + V\psi_\lambda}_{\lambda^2 T\psi + \lambda V\psi} \geq -AN$$

$\lambda \to 1-\lambda$

$$(1-\lambda)T\psi + V\psi \geq -\frac{AN}{1-\lambda} \quad //$$

$$T\psi = \frac{E}{\lambda} - \frac{1}{\lambda}\{(1-\lambda)T\psi + V\psi\}$$

$$\leq \frac{E}{\lambda} + \frac{AN}{\lambda(1-\lambda)}$$

$$= \frac{E+AN}{\lambda} + \frac{AN}{1-\lambda} \qquad \left(\frac{1}{\lambda} + \frac{1}{1-\lambda}\right)$$

$$= \left(\sqrt{E+AN} + \sqrt{AN}\right)^2 \quad \exists \lambda$$

$$\frac{X}{\lambda} + \frac{Y}{1-\lambda}$$

$$-\frac{X}{\lambda^2} + \frac{Y}{(1-\lambda)^2} = 0$$

$$\sqrt{Y}\,\lambda - \sqrt{X}\,(1-\lambda) = 0$$

$$(\sqrt{X}+\sqrt{Y})\lambda = \sqrt{X}$$

$$\lambda = \frac{\sqrt{X}}{\sqrt{X}+\sqrt{Y}}$$

$$\frac{X(\sqrt{X}+\sqrt{Y})}{\sqrt{X}} + \frac{Y(\sqrt{X}+\sqrt{Y})}{\sqrt{Y}} = (\sqrt{X}+\sqrt{Y})^2$$

$$\therefore K\int_{\mathbb{R}^3} \rho_\psi(x)^{\frac{5}{3}}\,dx \leq T\psi \leq \left(\sqrt{E+AN}+\sqrt{AN}\right)^2$$

$\forall p > 0$

$$\left(\int_{\mathbb{R}^3} \rho(x)^{\frac{5}{3}} dx\right)^{\frac{p}{2}} \int_{\mathbb{R}^3} |x|^p \rho(x)\, dx$$

$$\geq {}^{\exists}C_p \left(\int_{\mathbb{R}^3} \rho(x)\, dx\right)^{1+\frac{5}{6}p} \quad \text{---} \ (\star)$$

$$\left(\int |x|^p \rho_\psi(x)\, dx\right)^{\frac{1}{p}}$$

$$\geq C_p^{\frac{1}{p}} N^{\frac{1}{p}+\frac{5}{6}} \underbrace{\left(\int \rho(x)^{\frac{5}{3}} dx\right)^{-\frac{1}{2}}}_{\geq K^{\frac{1}{2}}\left(\sqrt{E+AN}+\sqrt{AN}\right)^{-1}}$$

$$\geq \underbrace{C_p^{\frac{1}{p}}}_{\gamma_p} K^{\frac{1}{2}} N^{\frac{1}{p}+\frac{5}{6}-\frac{1}{2}} \left(\sqrt{\frac{E}{N}+A}+\sqrt{A}\right)^{-1}$$

$$\therefore \left(\frac{1}{N}\int |x|^p \rho_\psi(x)\, dx\right)^{\frac{1}{p}}$$

$$\geq \gamma_p K^{\frac{1}{2}} N^{\frac{1}{3}} \left(\sqrt{\frac{E}{N}+A}+\sqrt{A}\right)^{-1} \quad //$$

$\boxed{\text{Fact}}$

$\forall \rho : \mathbb{R}^3 \longrightarrow [0, \infty)$

$$\left(\int_{\mathbb{R}^3} \rho(x)^{\frac{5}{3}} dx\right)^{\frac{p}{2}} \int_{\mathbb{R}^3} |x|^p \rho(x)\, dx$$

$$\geq \exists C_p \left(\int_{\mathbb{R}^3} \rho(x)\, dx\right)^{1+\frac{5}{6}p}$$

$$C_p = \left[\frac{4\pi}{p} B\left(\frac{7}{2}, \frac{3}{p}\right)\right]^{\frac{5}{6}p} \cdot \frac{\frac{5}{6}p}{\left(1+\frac{5}{6}p\right)^2}$$

$$\int_{\mathbb{R}^3} \rho(x)\,dx = \int_{|x|<1} \rho(x)\,dx + \int_{|x|>1} \rho(x)\,dx$$

$$\int_{|x|<1} \rho(x)\,dx$$
$$= \int (1-|x|^p)\rho(x)\,dx + \int |x|^p \rho(x)\,dx$$
$$\le \left(\int (1-|x|^p)^{\frac{5}{2}} dx\right)^{\frac{2}{5}} \left(\int \rho(x)^{\frac{5}{3}} dx\right)^{\frac{3}{5}}$$
$$+ \int_{|x|<1} |x|^p \rho(x)\,dx$$

$$\int_{|x|\gtrless 1} \rho(x)\,dx \le \int_{|x|>1} |x|^p \rho(x)\,dx$$

$$\therefore \int_{\mathbb{R}^3} \rho(x)\,dx$$
$$\le \underbrace{\left(\int_{|x|<1} (1-|x|^p)^{\frac{5}{2}} dx\right)^{\frac{2}{5}}}_{C_p} \left(\int_{\mathbb{R}^3} \rho(x)^{\frac{5}{3}} dx\right)^{\frac{3}{5}}$$
$$+ \int_{\mathbb{R}^3} |x|^p \rho(x)\,dx$$

$\rho(x) = (1-|x|^p)^{\frac{3}{2}}$

$$X \le C_p Y^{\frac{3}{5}} + Z$$

$$4\pi \int_0^1 (1-r^p)^{\frac{5}{2}} r^2\,dr$$
$$= \frac{4\pi}{p} B\left(\frac{7}{2}, \frac{3}{p}\right)$$

$$\Longrightarrow \quad Y^{\frac{P}{2}} Z \geq C X^{1+\frac{5}{6}P}$$

better

$$Y^{\frac{P}{2}}(X-C)^{\frac{3}{5}})$$

$\because)$ $\cancel{Y^{\frac{P}{2}} Z \geq Y^{\frac{P}{2}}}$

$$Y^{\frac{P}{2}} Z \geq \left\{C_p^{-1}(X-Z)\right\}^{\frac{5}{3}\cdot\frac{P}{2}} \cdot Z$$

$$= C_p^{\frac{5P}{6}} (X-Z)^{\frac{5}{6}P} Z$$

$$-\frac{5}{6}P(X-Z)^{-\frac{1}{6}P} Z + (X-Z)^{\frac{5}{6}P} = 0$$

$$(X-Z)^{-\frac{1}{6}P}\left\{-\frac{5}{6}PZ + (X-Z)\right\} = 0$$

$$\left(1+\frac{5}{6}P\right) Z = X$$

$$Z = \left(1+\frac{5}{6}P\right)^{-1} X$$

$$\therefore \quad Y^{\frac{P}{2}} Z \geq C_p^{\frac{5}{6}P}\left(1-\frac{1}{1+\frac{5}{6}P}\right) X^{\frac{5}{6}P}\left(1+\frac{5}{6}P\right)^{-1} X$$

$$= C_p^{\frac{5}{6}P} \frac{\frac{5}{6}P}{\left(1+\frac{5}{6}P\right)^2} X^{1+\frac{5}{6}P} \quad //$$

(ii)

$$\int_\Omega \rho_\psi(x)\,dx$$

$$= \int_\Omega 1 \cdot \rho_\psi(x)\,dx \qquad \text{Hölder}\left(\frac{2}{5}+\frac{3}{5}=1\right)$$

$$\leq \left(\int_\Omega 1\,dx\right)^{\frac{2}{5}} \left(\int_\Omega \rho_\psi(x)^{\frac{5}{3}}\,dx\right)^{\frac{3}{5}}$$

$$\leq \mathrm{vol}(\Omega)^{\frac{2}{5}} \left(\int_{\mathbb{R}^3} \rho_\psi(x)^{\frac{5}{3}}\,dx\right)^{\frac{3}{5}}$$

$$\therefore \frac{1}{N}\int_\Omega \rho_\psi(x)\,dx$$

$$\leq \frac{1}{N}\mathrm{vol}(\Omega)^{\frac{2}{5}} \cdot K^{-\frac{3}{5}} N^{\frac{3}{5}} \left(\sqrt{\frac{E}{N}+A}+\sqrt{A}\right)^{\frac{6}{5}}$$

$$= K^{-\frac{3}{5}} \left(\sqrt{\frac{E}{N}+A}+\sqrt{A}\right)^{\frac{6}{5}} \left(\frac{\mathrm{vol}(\Omega)}{N}\right)^{\frac{2}{5}}$$

§7.6 Instability for Bosons

7.6.1 The $N^{\frac{5}{3}}$ Law

$(\psi, H\psi)$

$$\gtrsim -0.231\,\alpha^2 q^{\frac{2}{3}} N \left(1 + 2.16\, Z \left(\frac{M}{N}\right)^{\frac{1}{3}}\right)^2$$

$$= -\,C\, N^{\frac{5}{3}} \quad (q = N)$$

$C \leftarrow Z,\ \frac{N}{M}$

Prop. 7.1 ($N^{\frac{5}{3}}$ Instability for Bosons)

$Z_j = Z \quad (j = 1, \dots, M)$

$\exists \psi \in \bigotimes^N L^2(\mathbb{R}^3)$ symmetric

$(\psi, \psi) = 1$

$\exists \underline{R} = (R_1, \dots, R_M) \in \mathbb{R}^{3M}$

s.t. $\mathcal{E}(\psi) = (\psi, H\psi)$

$$\leq -{}^{\exists}C\,\alpha^2 Z^{\frac{4}{3}} \min\{N, ZM\}^{\frac{5}{3}}$$

pf.

$$M = m^3 \quad (m \in \mathbb{Z})$$

$$N = ZM$$

$$\int |\phi_\lambda(x_i)|^2 dx_i = 1$$

$$\psi(x_1, \ldots, x_N) = \prod_{i=1}^{N} \phi_\lambda(x_i)$$

$$\phi_\lambda(x) = \lambda^{\frac{3}{2}} g(\lambda x)$$
$$(\lambda > 0)$$

$$g \in L^2(\mathbb{R}^3) \text{ s.t. } \int_{\mathbb{R}^3} |g(x)|^2 dx = 1$$

$$\begin{cases} (\psi, H\psi) = \frac{1}{2} N \lambda^2 \int_{\mathbb{R}^3} |\nabla g(x)|^2 dx + \lambda \alpha W(N, \underline{R}) \\ W(N, \underline{R}) = \frac{1}{2} N(N-1) \iint_{\mathbb{R}^3 \times \mathbb{R}^3} \frac{|g(x)|^2 |g(y)|^2}{|x-y|} dx dy \\ \qquad - ZN \sum_{k=1}^{M} \int_{\mathbb{R}^3} \frac{|g(x)|^2}{|x - R_k|} dx + U(\underline{R}) \end{cases}$$

$$\because) \ (\psi, -\frac{1}{2} \sum_{j=1}^{N} \Delta_j \psi) = \frac{1}{2} \sum_j \underbrace{\int_{\mathbb{R}^{3N}} |\nabla_j \psi(\underline{X})|^2 d\underline{X}}_{\int_{\mathbb{R}^3} |\nabla \phi_\lambda(x_j)|^2 dx_j}$$

$$\nabla \phi_\lambda(x) = \lambda^{\frac{3}{2}} \nabla(g(\lambda x)) = \lambda^{\frac{3}{2}+1} (\nabla g)(\lambda x)$$

$$= \frac{1}{2} N \lambda^2 \int_{\mathbb{R}^3} |(\nabla g)(\lambda x)|^2 \lambda^3 dx$$

$$|\nabla g(x)|^2 dx$$

$$\left(\psi, \sum_{i<j} \frac{1}{|x_i - x_j|} \psi\right)$$

$$= \lambda \sum_{i<j} \iint_{\mathbb{R}^3 \times \mathbb{R}^3} \frac{|g(\lambda x_i)|^2 |g(\lambda x_j)|^2}{|\lambda x_i - \lambda x_j|} \lambda^3 dx_i \cdot \lambda^3 dx_j$$

$$= \lambda \underbrace{\sum_{i<j}}_{\frac{1}{2} N(N-1)} \iint \frac{|g(x)|^2 |g(y)|^2}{|x-y|} dx\, dy$$

$$\left(\psi, \sum_{j=1}^{N} \sum_{k=1}^{M} \frac{Z}{|x_j - R_k|} \psi\right)$$

$$= \lambda Z \sum_{j} \sum_{k} \int_{\mathbb{R}^3} \frac{|g(\lambda x_j)|^2}{|\lambda x_j - \lambda R_k|} \lambda^3 dx_j$$

$$= \lambda Z \underbrace{\sum_{j}}_{N} \sum_{k} \int \frac{|g(x)|^2}{|x - R_k|} dx$$

$$(\psi, U(R)\psi)$$

$$= (\psi, \psi) \sum_{k<l} \frac{Z^2}{|\tilde{R}_k - \tilde{R}_l|} = \underbrace{\lambda Z^2 \sum_{k<l} \frac{1}{|R_k - R_l|}}_{U(R)} \quad //$$

cont...

$$W(N,\underline{B}) \le -\underbrace{{}^{\exists}C Z^{\frac{2}{3}} N^{\frac{4}{3}}}_{B} \quad \text{——} \ (\star)$$

$$\Longrightarrow (\psi, H\psi) \le A\lambda^2 - B\lambda$$

$$= A\left(\lambda - \frac{B}{2A}\right)^2 - \frac{B^2}{2A}$$

$$= -\frac{B^2}{2A} \quad ({}^{\exists}\lambda)$$

$$= -\frac{C'\alpha^2 Z^{\frac{4}{3}} N^{\cancel{\frac{8}{3}}\ \frac{5}{3}}}{\cancel{N}\int |\nabla g(x)|^2 dx}$$

$\bigcup_k \Gamma_k = \operatorname{supp} g \subset\subset \mathbb{R}^3$

s.t. $\int_{\Gamma_k} |g(x)|^2 dx = \frac{1}{M} \quad (k = 1, \dots, M)$

$R_k \in \Gamma_k$

avg. value with weight $M|g(x)|^2$ in Γ_k.

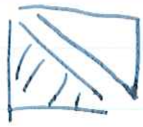
$$-ZN \sum_k \int_{\Gamma_k} \int_{\mathbb{R}^3} \frac{|g(x)|^2}{|x - \underset{y}{\cancel{R_k}}|} dx \cdot M|g(y)|^2 dy$$

$$= -ZNM \underbrace{\sum_k \int_{\Gamma_k}}_{\int_{\bigcup_k \Gamma_k} \Rightarrow \int_{\mathbb{R}^3}} \int_{\mathbb{R}^3} \frac{|g(x)|^2 \cdot |g(y)|^2}{|x-y|} dx dy$$

$$\iint \underbrace{U(\underline{R})}_{\substack{\displaystyle \sum_{k<l} \frac{Z^2}{|x-y|}}} M|g(x)|^2 dx \cdot M|g(y)|^2 dy$$

$$= Z^2 M^2 \sum_k \int_{\Gamma_k} \sum_l \int_{\Gamma_l} \frac{|g(x)|^2 \cdot |g(y)|^2}{|x-y|} dx\, dy$$

$$\sum_{k<l} = \frac{1}{2}\left(\sum_k \sum_l - \sum_k \sum_k \right)$$



$$= \frac{1}{2} Z^2 M^2 \iint_{\mathbb{R}^3 \times \mathbb{R}^3} \frac{|g(x)|^2 \cdot |g(y)|^2}{|x-y|} dx\, dy$$

$$- \frac{1}{2} Z^2 M^2 \sum_k \iint_{\Gamma_k \times \Gamma_k} \frac{|g(x)|^2 \cdot |g(y)|^2}{|x-y|} dx\, dy$$

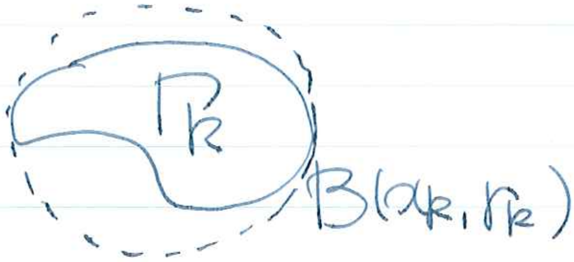
$\exists \underline{R}$ s.t. $W(N, \underline{R}) \leq \langle W(N, \underline{R}) \rangle_{\underline{R}}$

$$= \Big(\underbrace{\frac{1}{2} N(N-1) - ZNM + \frac{1}{2} Z^2 M^2}_{\leq 0 \text{ if } N = ZM} \Big) \iint \frac{|g(x)|^2 \cdot |g(y)|^2}{|x-y|} dx\, dy$$

$$- \frac{1}{2} Z^2 M^2 \sum_k \iint_{\Gamma_k \times \Gamma_k} \frac{|g(x)|^2 \cdot |g(y)|^2}{|x-y|} dx\, dy$$

$$\leq - \frac{1}{2} Z^2 M^2 \sum_k \iint_{\Gamma_k \times \Gamma_k} \frac{|g(x)|^2 \cdot |g(y)|^2}{|x-y|} dx\, dy$$

Take the smallest $r_k > 0$ s.t. $\Gamma_k \subset B(x_k, r_k)$



$$\frac{1}{2}\iint_{\Gamma_k\times\Gamma_k} \frac{|g(x)|^2|g(y)|^2}{|x-y|}\,dx\,dy \geq \left(\frac{1}{M}\right)^2 \frac{1}{2r_k}$$

$$\frac{1}{2M^2 r_k}$$

Jensen's inequality

f: convex, $\mu > 0$ on $\mathbb{R}$

$$\Longrightarrow \frac{1}{\mu(\mathbb{R})}\int f(x)\,\mu(dx) \geq f\left(\frac{\int x\,\mu(dx)}{\mu(\mathbb{R})}\right)$$

$$f(x) = \frac{1}{x} \quad (x > 0)$$

$$\mu(dx) = \sum_{j=1}^{M} \delta(x - r_j)\,dx, \quad \mu(\mathbb{R}) = \sum_j 1 = M$$

$$\frac{1}{M}\underbrace{\int \frac{1}{x}\,\mu(dx)}_{\sum_k \frac{1}{r_k}} \geq \frac{M}{\underbrace{\int x\,\mu(dx)}_{\sum_k r_k}}$$

$$\frac{1}{2} Z^2 M^2 \sum_k \iint_{\Gamma_k \times \Gamma_k} \frac{|g(x)|^2 \cdot |g(y)|^2}{|x-y|} \, dx\, dy$$

$$\geq \frac{1}{2} Z^2 M^2 \sum_k \frac{1}{M^2 r_k}$$

$$= \frac{1}{2} Z^2 M \cdot M^{-1} \sum_k \frac{1}{r_k}$$

$$\geq \frac{1}{2} Z^2 M \cdot \frac{1}{\frac{1}{M} \sum_k r_k}$$

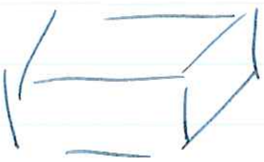
$$x = (x^1, x^2, x^3) \in \mathbb{R}^3$$

$$g(x) = f(x^1) f(x^2) f(x^3)$$

$$\mathrm{supp.}\, f \subset [-1, 1] = \bigcup_j I_j$$

$$\int_{I_j} |f(t)|^2 dt = \frac{1}{n} \quad (j = 1, \dots, n)$$

$$\Gamma_k = I_{j1} \times I_{j2} \times I_{j3}$$

 $|\Gamma_k| = s \cdot t \cdot u$

$$r_k = \frac{1}{2}(s^2+t^2+u^2)^{\frac{1}{2}}$$
$$\leq \frac{1}{2}(s+t+u)$$

$$\therefore \frac{1}{M}\sum_{k=1}^{M} r_k \leq \frac{1}{2}\left(\frac{1}{m}+\frac{1}{m}+\frac{1}{m}\right)$$
$$= \frac{3}{2}\frac{1}{m} = \frac{3}{2}M^{-\frac{1}{3}}$$

$\therefore (\psi, H\psi)$

$$\leq -\frac{1}{2}Z^2 M \frac{1}{\frac{1}{M}\sum_k r_k}$$
$$\leq -\frac{1}{2}Z^2 M \frac{1}{\frac{3}{2}M^{-\frac{1}{3}}} = -\frac{1}{3}Z^2 M^{\frac{4}{3}}$$
$$Z^{\frac{2}{3}}(\underbrace{ZM}_{N})^{\frac{4}{3}}$$

//

(☆)