$$\Gamma\left(\frac{d}{2}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)=\frac{1}{2^{d-1}}\Gamma(d)\Gamma\left(\frac{1}{2}\right)$$

for $\forall d = 1, 2, 3, \ldots$

Proof.

@ $d = 2k+1$

$$\Gamma\left(\frac{d}{2}+\frac{1}{2}\right)=\Gamma(k+1)=k!=\frac{(2k)!!}{2^k}$$

$$\Gamma\left(\frac{d}{2}\right)=\Gamma\left(k+\frac{1}{2}\right)$$

$$=\left(k-\frac{1}{2}\right)\left(k-\frac{3}{2}\right)\cdots\left(k-\frac{2k-1}{2}\right)\Gamma\left(\frac{1}{2}\right)$$

$$=\frac{(2k-1)!!}{2^k}\Gamma\left(\frac{1}{2}\right)$$

$$\therefore \Gamma\left(\frac{d}{2}+\frac{1}{2}\right)\Gamma\left(\frac{d}{2}\right)=\frac{(2k)!}{2^{2k}}\Gamma\left(\frac{1}{2}\right)$$

$$=\frac{1}{2^{d-1}}\Gamma(d)\Gamma\left(\frac{1}{2}\right) \quad //$$

@ $d = 2k$

$$\begin{cases}\Gamma\left(\frac{d}{2}+\frac{1}{2}\right)=\frac{(2k-1)!!}{2^k}\Gamma\left(\frac{1}{2}\right)\\ \Gamma\left(\frac{d}{2}\right)=\frac{(2k-2)!!}{2^{k-1}}\end{cases} \Rightarrow (\mathrm{RHS}) \quad //$$

Q.E.D.