$$r_j^{(M)} = f^{(M)}_{\sigma_{M-1}\cdots\sigma_1(j)}$$

$$r_j^{(M)} \longrightarrow \exists r_j \quad (M\to\infty)$$

$$r_j^{(M)}\Big(1-\sum_{k=1}^{M} c_k\Big) = R_j^{(M)}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\underbrace{1-d \qquad\qquad R_j}$$

$$\Rightarrow r_j = \frac{R_j}{1-d}$$

$\forall \epsilon > 0 \ \exists M_0 \in \mathbb{N}$

s.t. $M \geq M_0 \Rightarrow |r_j^{(M)} - r_j| < \epsilon$

$$\epsilon = \frac{r_j}{2} \Rightarrow 0 \leq \frac{r_j}{2} < r_j^{(M)} \quad (\forall j : \text{fixed})$$

$$r_j^{(M)} \xmapsto{\tau_M} \tilde{r}_j^{(M)} \quad \text{s.t. } \tilde{r}_j^{(M)} = r^{(M)}_{\tau_M(j)}$$

$$0 \leq \frac{r_{\tau_M(j)}}{2} < r^{(M)}_{\tau_M(j)} = \tilde{r}_j^{(M)}$$

$$\therefore \frac{r_{\tau_M(N)}}{2} < \tilde{r}_N^{(M)} \qquad r_{\tau_M(N)} \neq 0$$

$$\varepsilon_M = \min\left\{ \tilde{r}_N^{(M)},\ 1 - \tilde{r}_{N+1}^{(M)} \right\}$$
$$\geq \min\left\{ \frac{r_{\tau_M(N)}}{2},\ \frac{1}{N+1} \right\}$$

$\forall M \geq M_0$ $\qquad r_{\tau_M(N)} > 0$

$$r \in \mathcal{F}$$
$$\downarrow$$
$$\tilde{r} \in \tilde{\mathcal{F}} \Rightarrow \tilde{r}_N > 0$$

$$0 < \frac{\tilde{r}_N}{2} < \tilde{r}_N^{(M)}$$

$$0 \leq \frac{r_j}{2} < r_j^{(M)} \quad (\forall j : \text{fixed})$$

$$\downarrow \tau$$

$$0 \leq \frac{\widehat{r}_j}{2} < \widehat{r}_j^{(M)}$$

$$\varepsilon_M = \min\left\{ \widehat{r}_N^{(M)},\ 1 - \widehat{r}_{N+1}^{(M)} \right\}$$

$$> \min\left\{ \frac{\widehat{r}_N}{2},\ \frac{1}{N+1} \right\}$$

On the other hand,

$$\widehat{r} \in \widehat{\mathcal{F}} \Rightarrow \widehat{r}_N > 0$$

$$\forall M \geq M_0 \ \exists \varepsilon_\infty > 0$$

$$\text{s.t.} \quad \varepsilon_M \geq \varepsilon_\infty > 0 \quad //$$